International Conference on Diophantine Analysis and related Topics

Wuhan University
Wuhan, Hubei Province, China

March 10–13, 2016

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Conference venue
School of Mathematics and Statistics, Wuhan University, 430072, P.R. China

Conference hotels
• Luo Jia Shan Zhuang
Next to the Wuhan University Administration Building, Wuchang Bayi Road, Wuchang District, Wuhan 430070, China
Tel: +86-(0)27-68752935
• Fengyi Hotel
No.336 Bayi Road, Hongshan District, Wuhan 430072, China
Tel: +86-400-875-8866
Conference hotels are available only from Wednesday, March 9 (after 14:00) to Monday, March 14 (until 12:00 noon).

Registration
The registration will be held from 14:00 to 18:00 on March 9 at the lobby of the hotels (Luo Jia Shan Zhuang and Fengyi Hotel, respectively), and from 8:30 to 9:00 or break time at the lobby of Math Building of Wuhan University on March 10.

Meal
Breakfast is included in the hotel rate.
Lunch will be taken at the 1st floor of **Guiyuan Can Ting** (Restaurant) near Luo Jia Shan Zhuang.
Dinner will be taken at **Jia Yuan Xiao Guan Yuan** (Restaurant) near Fengyi Hotel and the entrance of the tunnel.

Banquet
The banquet is only for invited speakers, and at the 2nd floor at Luo Jia Shan Zhuang from 18:00 on Friday, March 11.

Contact information
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Program

Thursday, March 10

9:00–9:20	Opening
	Moderator: István Mező
9:20–10:00	Roberto B. Corcino (Cebu Normal Univ.) The Hankel transform of q-noncentral Bell numbers
10:00–10:30	Cristina B. Corcino (Cebu Normal Univ.) Asymptotic formulas for r-Stirling numbers of the first kind
	Break
	Moderator: Gopal Krishna Panda
11:00–11:30	Tuangrat Chaichana (Chulalongkorn Univ.) An approximation property of lacunary power series in the field of formal series
11:30–12:00	Jittinart Rattanamoong (Srinakharinwirot Univ.) Independence of some special series
	Lunch
	Moderator: Wen Huang
14:20–15:00	Masanobu Kaneko (Kyushu Univ.) Finite multiple zeta values
15:00–15:40	Paul Thomas Young (College of Charleston) Bernoulli polynomial convolutions and p-adic Arakawa-Kaneko zeta functions
	Break

Moderator: Abdelmejid Bayad

16:10–16:50 Genki Shibukawa (Osaka Univ.)
A multivariate analogue of the Cauchy numbers

16:50–17:30 Yilmaz Simsek (Akdeniz Univ.)
A note on combinatorial sums

Dinner

Friday, March 11

Moderator: Yilmaz Simsek

9:00–9:40 Abdelmejid Bayad (Univ. d'Evry Val d'Essonne)

Arithmetic of Jacobi modular forms

9:40–10:10 Miho Aoki (Shimane Univ.)
On equivalence classes of generalized Fibonacci sequences associated to units of real quadratic fields

Break

Moderator: Florian Luca

10:40–11:20 Zhi-Wei Sun (Nanjing Univ.)
Some new diophantine problems

11:20–11:50 Prasanta Kumar Ray (Veer Surendra Sai Univ. of Tech.)
Certain Diophantine equations involving balancing and Lucas-balancing numbers

Lunch

Free discussion

Saturday, March 12

Moderator: László Szalay

9:00–9:40	Attila Bérczes (Univ. Debrecen) Effective results for Diophantine equations over finitely generated domains
9:40–10:10	András Bazsó (Univ. Debrecen) Diophantine equations concerning Appell sequences
	Break

Moderator: Masanobu Kaneko

10:40–11:20	Pingzhi Yuan (South China Normal Univ.) Ke Zhao's method and its applications to Diophantine equations
11:20–12:00	Lajos Hajdu (Univ. Debrecen) Perfect powers in a family of combinatorial polynomials
	Group photo
	Lunch

Moderator: Paul Thomas Young

14:20–15:00	Simon Kristensen (Aarhus Univ.) Orbits of rotations and beyond
15:00–15:40	Shuai Zhai (Shandong Univ.) On the 2-part of the Birch and Swinnerton-Dyer conjecture for quadratic twists of elliptic curves
	Break

Moderator: Zhi-Wei Sun

16:10–16:50 Florian Luca (Wits Univ.)
Diophantine m-tuples with values in linear recurrences

16:50–17:30 Christian Ballot (Univ. Caen)
On recurrence-based numerations

Dinner

Sunday, March 13

Moderator: Lajos Hajdu

9:00–9:40 Wen Huang (Sichuan Univ. and USTC)
Common difference sets, higher order recurrence sets and related problems

9:40–10:10 Oliver Roche-Newton (Wuhan Univ.)
Extremal problems for distances and sum-products

Break

Moderator: Pingzhi Yuan

10:40–11:20 Lilu Zhao (Hefei Univ. Tech)
Minor arcs estimate in the application of circle method

11:20–12:00 Yuchao Wang (Shanghai Univ.)
On the saturation number for cubic surfaces

Lunch

Moderator: Attila Bérczes

14:20–15:00	Isao Wakabayashi (Seikei Univ.) Number of solutions for sextic simple Thue equations
15:00–15:40	Gopal Krishna Panda (National Institute of Tech., Rourkela) Sums and reciprocal sums involving balancing numbers
	Break

Moderator: Roberto B. Corcino

16:10–16:50	László Szalay (Univ. West Hungary) Pascal-like arrays and triangles
16:50–17:30	István Mező (NUIST) Partitions without small blocks
	Closing
	Dinner

The Hankel transform of q-noncentral Bell numbers

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Abstract

The Hankel transform of an integer sequence (a_n) is defined as a sequence formed by the determinants of the matrices A_n, where A_n is the upper-left submatrix of size $n \times n$ of the Hankel matrix

$$A = \begin{pmatrix} a_0 & a_1 & a_2 & a_3 & \cdots \\ a_1 & a_2 & a_3 & a_4 & \cdots \\ a_2 & a_3 & a_4 & a_5 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}.$$

In this talk, methods in deriving the Hankel transform of Bell and other Bell-type numbers will be discussed, particularly, in using Layman's Theorem on the invariance of Hankel transform under binomial transform. Moreover, the Hankel transform of the q-analogue of non-central Bell numbers will be derived using Spivey-Steil's Theorem on the Hankel transform of rising k-binomial transforms.

Keywords: Hankel matrix; Hankel determinant; Hankel transform; Stirling numbers; Bell numbers; binomial transform;

References

[1] M. Aigner, *A characterization of the Bell numbers*, Discrete Math. **205** (1999), 207–210.

[2] A.Z. Broder, em The r-Stirling numbers, Discrete Math. **49** (1984), 241–259.

[3] J. Cigler, *Hankel determinants of q-exponential polynomials*, arXiv:0901.4698.

[4] J. Cigler, *Hankel determinants of generalized q-exponential polynomials*, arXiv:0909.5581.

[5] J. Cigler, *Eine Charakterisierung der q- Exponentialpolynome*, Sitzber. OeAW **208** (1999), 143–157.

[6] L. Comtet, *Advanced Combinatorics*, Reidel, Dordrecht, The Netherlands, 1974.

[7] K. Conrad, *A q-analogue of Mahler expansions I*, Adv. in Math. **153** (2000), 185–230.

[8] R. B. Corcino and C. B. Corcino, *The Hankel transform of generalized Bell numbers and its q-analogue*, Utilitas Mathematica **89** (2012), 297–309.

[9] A. Cvetković, P. Rajković and M. Ivković, *Catalan numbers, the Hankel transform and Fibonnaci numbers*, J. Integer Seq. **5** (2002), Article 02.1.3.

[10] M. Desainte-Catherine and X. G. Viennot, *Enumeration of certain Young tableaux with bound height*, Combinatorie Énumérative (Montreal 1985), Lect. Notes in Math. **1234** (1986), 58–67.

[11] R. Ehrenborg, *Determinants involving q-Stirling numbers*, Adv. in Appl. Math. **31** (2003), 630–642.

[12] M. Garcia-Armas and B. A. Seturaman, *A note on the Hankel transform of the central binomial coefficients*, J. Integer Seq. **11** (2008), Article 08.5.8.

[13] H.W. Gould, *The q-Stirling number of the first and second kinds*, Duke Math. J. **28** (1968) 281–289.

[14] M. S. Kim and J. W. Son, *A note on q-difference operators*, Commun. Korean Math. Soc. **17** (2002), No. 3, pp. 423–430.

[15] M. Koutras. em Non-central Stirling numbers and some applications, Discrete Math. **42** (1982), 73–89.

[16] J.W. Layman, *The Hankel transform and some of its properties*, J. Integer Seq. **4** (2001), Article 01.1.5.

[17] A. De Medicis and P. Leroux, *Generalized Stirling numbers, convolution formulae and p, q-analogues*, Can. J. Math **47**(3) (1995), 474–499.

[18] I. Mező, *The r-Bell numbers*, J. Integer Seq. **14** (2011), Article 11.1.1.

[19] C. Radoux, *Déterminat de Hankel construit sur des polynomes liés aux nombres de dérangements*, European Journal of Combinatorics **12** (1991) 327–329.

[20] J. Riordan, *Combinatorial identities*, Wiley, New York, 1968.

[21] N. J. Sloane, *The On-line Encyclopedia of Integer Sequences*, available at http://oeis.org.

[22] M. Z. Spivey and L. L. Steil, *The k-binomial transform and the Hankel transform*, J. Integer Sq. **9** (2006), Article 06.1.1.

[23] U. Tamm, *Some aspects of Hankel matrices in coding theory and combinatorics*, Electron. J. combin. **8**(1) (2001), # A1.

[24] R. Vein and A. Dale, *Determinants and their applications in Mathematical Physics,* Springer, 1991.

[25] D. Zelinsky, *A first course in Linear Algebra*, 2ed, Academic Press, Inc.,1973.

Asymptotic formulas for r-Stirling numbers of the first kind

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Abstract

The r-Stirling numbers of the first kind count the number of permutations of the set $\{1, 2, \ldots, n\}$ with m cycles such that the first r elements are in distinct cycles. These numbers were first introduced by Andrei Broder in his paper 'The r-Stirling Numbers' published in Discrete Mathematics. The r-Stirling numbers of the second kind were also studied in the paper by Broder but focus here will be on the first kind only.

Andrei Broder denoted the r-Stirling numbers of the first kind by $\left[{n \atop m} \right]_r$. Since $\left[{n \atop m} \right]_r = 0$ for $m < r$, this study considers the r-Stirling numbers of the first kind $\left[{n+r \atop m+r} \right]_r$, where n, m, r are positive integers. These numbers satisfy the relation

$$z(z+1)(z+2)\ldots(z+n-1) = \sum_{m=0}^{n} \left[{n+r \atop m+r} \right]_r (z-r)^m.$$

In this presentation an asymptotic formula for r-Stirling numbers of the first kind obtained using the Moser-Wyman method will be discussed and will be shown to be asymptotically equivalent with another asymptotic formula obtained using a modified saddle-point method. Moreover, it will be shown that the formulas can be extended to r-Stirling type numbers of the first kind.

Keywords: r-Stirling numbers; asymptotic analysis;asymptotic approximation

References

[1] M. Abramowitz and I. A. Stegun, *Hanbook of Mathematical Functions*, Dover Publications, Inc., 1972.

[2] A.Z. Broder, *The r-Stirling Numbers*, Discrete Math. **49**(1984), 241–259.

[3] C. B. Corcino, L.C. Hsu and E.L. Tan, *Asymptotic approximations of r-Stirling numbers*, Approximation Theory and its Applications **15:3** (1999), pp. 13–25.

[4] E. T. Copson, *Asymptotic Expansions*, Cambridge University Press, 1967.

[5] R. Chelluri, L. B. Richmond and N. M. Temme, *Asymptotic Estimates for Generalized Stirling Numbers*, Analysis **20** (2000), 1–13.

[6] J. B. Conway, *Functions of one complex variable*, 2nd ed. Springer Verlag, New York, 1978.

[7] L. C. Hsu and P. J-S. Shuie, *A unified approach to generalized Stirling numbers*, Advances in Applied Mathematics (1998), pp. 366–384.

[8] L. Moser and M. Wyman, *Asymptotic development of the Stirling numbers of the first kind*, J. London Math. Soc. **33** (1958), 133–146.

[9] N. M. Temme, *Asymptotic estimates of Stirling numbers*, Studies in Applied Mathematics **89**, (1993), pp. 233–243.

[10] M. A. R. P Vega and C. B. Corcino, *An asymptotic formula of the generalized Stirling numbers of the first kind*, Util. Math. **73** (2007), 129–141.

An approximation property of lacunary power series in the field of formal series

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Abstract

Let $\mathbb{F}((1/x))$ be the field of formal series over a field $\mathbb{F}$ complete with respective to the degree valuation $|\cdot|$. It is shown that for any positive number ω, there is no lacunary power series $f(z) \in \mathbb{F}(x)[[z]]$ such that

$$f(\mathbb{F}(x)) \subseteq \mathbb{F}(x) \quad \text{and} \quad |\text{den } f(p/q)| << |q|^{\omega}$$

for all $p,\ q \in \mathbb{F}[x]$, with q being of sufficiently large degree.

Keywords: lacunary power series

References

[1] K. Mahler, *Some suggestions for further research*, Bull. Austral. Math. Soc. **29** (1984), 101-108.

[2] D. Marques, J. Ramirez and E. Silva, *A note on lacunary power series with rational coefficients*. Bull. Austral. Math. Soc., available on CJO2015. doi:10.1017/S0004972715001239.

Independence of some special series

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Abstract

Consider those real numbers whose infinite series representations are

$$\alpha_j := \sum_{k=1}^{\infty} \frac{j^k}{g_1 g_2 \cdots g_k} \qquad (1 \le j \le n)\,,$$

where $\{g_r\} = \{g_1 = 1, g_2, g_3, \ldots\}$ is a strictly increasing sequence of natural numbers. Such series were used by Hlawka ([1]) in his work on applications of diophantine approximation to problems in differential equations. Hlawka proved certain independence results about these numbers. Here, we derive criteria for algebraic and linear independence of these numbers extending those of Hlawka.

Keywords: algebraic and linear independence

References

[1] E. Hlawka, *Eine Anwendung diophantischer Approximationen auf die Theorie von Differentialgleichungen*, Aequationes Math. **35** (1988), 232–253.

Finite multiple zeta values

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Abstract

We discuss two very different "finite" versions of the classical multiple zeta values

$$\zeta(k_1,\ldots,k_r) \;=\; \sum_{0<n_1<\cdots<n_r} \frac{1}{n_1^{k_1}\cdots n_r^{k_r}} \in \mathbb{R} \quad (k_1,\ldots,k_r\in\mathbb{N}),$$

which are very interesting real numbers if $k_r \geq 2$ but are infinite if $k_r = 1$. One of these is the element $\zeta^{\mathcal{A}}(\mathbf{k})$ of the "poor man's adele ring"

$$\mathcal{A} \;:=\; \prod_{p \text{ prime}} \mathbb{Z}/p\mathbb{Z} \Big/ \bigoplus_{p \text{ prime}} \mathbb{Z}/p\mathbb{Z}$$

defined by $\zeta^{\mathcal{A}}(k_1,\ldots,k_r) \;=\; \big(\zeta_p(k_1,\ldots,k_r) \bmod p\big)_{p \text{ prime}}$, where

$$\zeta_p(k_1,\ldots,k_r) \;=\; \sum_{0<n_1<\cdots<n_r<p} \frac{1}{n_1^{k_1}\cdots n_r^{k_r}}\,.$$

The other is the *symmetric multiple zeta value* or *finite real multiple zeta value* defined by

$$\zeta^{\mathcal{S}}(k_1,\ldots,k_r) \;=\; \sum_{i=0}^{r}(-1)^{k_{i+1}+\cdots+k_r}\zeta(k_1,\ldots,k_i)\,\zeta(k_r,\ldots,k_{i+1})\,,$$

which is obviously finite if all k_i are ≥ 2 but turns out also to be finite if the k_i are arbitrary positive integers and the multiple zeta values appearing in the right-hand side are replaced by their regularized values. (More precisely, we consider this value in the quotient ring of the multiple zeta values modulo π^2.)

After giving some basics, we present a conjectural isomorphism of the two rings of finite multiple zeta values and give some results which support our conjecture. This is a joint work with Don Zagier.

Keywords: (finite) multiple zeta values, regularization

Bernoulli polynomial convolutions and p-adic Arakawa-Kaneko zeta functions

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Abstract

We evaluate the ordinary convolution of Bernoulli polynomials in closed form in terms of poly-Bernoulli polynomials. As applications we derive identities for p-adic Arakawa-Kaneko zeta functions, including a p-adic analogue of Ohno's sum formula. These p-adic identities serve to illustrate the relationships between real periods and their p-adic analogues.

Keywords: Bernoulli polynomials; convolution identities; poly-Bernoulli polynomials; Arakawa-Kaneko zeta functions; p-adic analysis.

References

[1] P.T. Young, *The p-adic Arakawa-Kaneko zeta functions and p-adic Lerch transcendent*, J. Number Theory **155** (2015), 13–55.

[2] P. T. Young, *Bernoulli and poly-Bernoulli polynomial convolutions and identities of p-adic Arakawa-Kaneko zeta functions*, Int. J. Number Theory, to appear.

A multivariate analogue of the Cauchy numbers

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Abstract

We introduce a multivariate analogue of the Cauchy numbers of the first kind (the Bernoulli numbers of the second kind) defined by a definite integral of the shifted Schur functions which are regarded as a multivariate analogue of the falling factorials. We give their fundamental properties ; the explicit expressions by a multivariate analogue of the Stirling numbers, generating function and convolution identities. They are analogous to those of the classical Cauchy numbers. We also give a new zeta function associated with our Cauchy numbers.

Keywords: shifted Schur functions; Starling numbers; Cauchy numbers.

References

[1] I. G. Macdonald, *Symmetric Functions and Hall Polynomials*, Oxford University Press, 1995.

[2] A. Okounkov and G. Olshanski, *Shifted Schur Functions*, Algebra i Analiz, **9-2** (1998), 73–146 (Russian); English version in St. Petersburg Math. J. **9-2** (1998), 239–300.

A note on combinatorial sums

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Abstract

The aim of this talk is to firstly give some identities of the Bernstein basis functions and their generating functions. Secondly, integrating these identities, we give combinatorial sums involving binomial coefficients, Pascal's rule, Vandermonde's type of convolution, the Bernoulli polynomials. Finally, we give some remarks and comments on these combinatorial sums.

Keywords: Bernstein basis functions, generating functions, combinatorial sums, Pascal's rule, Vandermonde's type of convolution

References

[1] L. Comtet, Advanced Combinatorics: The Art of Finite and Infinite Expansions (Translated from the French by J. M. Nienhuys), Reidel: Dordrecht and Boston, 1974.

[2] H. W. Gould, Combinatorial Identities Vol.1–Vol.8. http://www.math.wvu.edu/ gould/.

[3] Y, Simsek, *Combinatorial Identities associated with Bernstein type basis functions* , to appear Filomat (2015).

[4] Y. Simsek, *Analysis of the Bernstein basis functions: an approach to combinatorial sums involving binomial coefficients and Catalan numbers*, Math. Meth. Appl. Sci. **38** (2015), 3007–3021.

[5] Y. Simsek, *Beta-type polynomials and their generating functions*, Appl. Math. Comput. **254** (2015) 172–182

[6] R. P. Stanley, *Enumerative Combinatorics*, Vol. **1–2**. Cambridge University Press: Cambridge, 1999.

[7] B.Sury, *Sum of the reciprocals of the binomial coeffcients*. European Journal of Combinatorics **14** (1993), 351–353.

Arithmetic of Jacobi modular forms

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Abstract

In this Talk we study certain Jacobi modular form D_L. We give its relation to theta function of genus one and its link with Eisenstein series, and also its relationship with Kronecker's series. By the logarithmic derivative, we derive from it an integral representative of D_L. We consider the Laurent series coefficients of D_L and establish a recurrence formula for them. Those coefficients are elliptic analogue to periodized Bernoulli functions. Several interesting identities can be obtained from our study. These interrelations were already the object of the Weil's book [7]. New relations are here obtained.

Keywords: Jacobi form; Theta function; Eisenstein series; Kronecker series, Elliptic Bernoulli functions.

References

[1] A. Bayad, *Jacobi forms in two variables: Analytic theory and elliptic Dedekind sums,* (2011).

[2] A. Bayad and G. Robert, *Note sur une forme de Jacobi méromorphe*, C.R.A.S Paris, Ser. I **325**, (1997), 455-460.

[3] H. Ito, *A function on the upper half space which is analogous to the imaginary part of* $\log \eta(z)$, J. reine angew. Math. **373** (1986), 148–165.

[4] K. Katayama, *On the values of Eisenstein series,* Tokyo J. of Math. **1**, (1978), 157–188.

[5] R. Sczech, *Dedekindsummen mit elliptischen Funktionen,* Inventiones math. **76**, (1984), 523–551.

[6] C. L. Siegel, *Advanced analytic number theory*, Tata Institute , Bombay India (Edition 1980).

[7] A. Weil, *Elliptic functions according to Eisenstein and Kronecker*, (Ergeb. der Math. 88), Springer-Verlag, (1976).

[8] A. Weil, *Introduction à l'étude des variétés Kählériennes*, Paris, Hermann, (1958).

On equivalence classes of generalized Fibonacci sequences associated to units of real quadratic fields

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Abstract

Let F be a real quadratic field, and α ($\neq \pm 1$) be a unit of F. Let $t = \mathrm{Tr}(\alpha)$ and $u = N(\alpha)$ denote the trace and the norm of α, respectively. We call $\{G_n\}$ a generalized Fibonacci sequence associated to α given by $G_n = tG_{n-1} - uG_{n-2}$ for arbitrary first and second terms $G_1, G_2 \in \mathbb{Z}$. If $G_1 = 1$ and $G_2 = t$, then we denote the sequence by $\mathcal{F}_n$. We fix a prime number p and introduce an equivalence relation $\sim^*$ for the set of generalized Fibonacci sequences associated to α; $\{G_n\} \sim^* \{G'_n\}$ if there are some integers m and n satisfying $G_{n+1}G'_n \equiv G'_{n+1}G_m \pmod{p}$. We determine the number of equivalence classes $\overline{\{G_n\}}$ satisfying $p \nmid G_n$ for any integers n. From our results, for example, we can know the following. (1) If $\left(\dfrac{d}{p}\right) = 1$ or 0 , there are infinitely many generalized Fibonacci sequences $\{G_n\}$ associated to α that satisfy $p \nmid G_n$ for any $n \in \mathbb{Z}$. (2) If $\left(\dfrac{d}{p}\right) = -1$ and $d(p) = p + 1$, then for any generalized Fibonacci sequences $\{G_n\}$ associated to α, we have $p | G_n$ for some $n \in \mathbb{Z}$, where d is the discriminant and $d(p)$ is the smallest positive integer n that divides $\mathcal{F}_n$.

Keywords: generalized Fibonacci sequences; real quadratic fields.

Some new diophantine problems

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Abstract

In this talk we introduce various new diophantine problems and related conjectures made by the speaker. For example, the speaker conjectured that any positive rational number can be written as a sum of finitely many distinct unit fractions of the form $1/(p-1)$ (or $1/(p+1)$) with p prime, and that any natural number can be written as the sum of a nonnegative cube, a square and a triangular number. The problems might interest number theorists and stimulate further research.

Keywords: Conjectures; diophantine equations; representations; primes.

References

[1] Z.-W. Sun, *On universal sums of polygonal numbers*, Sci. China Math. **58** (2015), 1367-1396.

[2] Z.-W. Sun, *Conjectures on representations involving primes*, in: Combinatorial and Additive Number Theory (edited by M.B. Nathanson), Springer, to appear.

Certain Diophantine equations involving balancing and Lucas-balancing numbers

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Abstract

Balancing numbers are originally introduced by Behera and Panda as solution of a Diophantine equation on triangular numbers. They found that a natural number x is a balancing number if and only if $8x^2+1$ is a perfect square. Subsequently, in a latter paper, Panda call the positive square root of $8x^2+1$ as a Lucas-balancing number. Thus, the totality of balancing number x and Lucas-balancing number y are seen to be the positive integral solutions of the Diophantine equation $8x^2+1 = y^2$. It is observed that the Lucas-balancing numbers are associated with balancing numbers in the way Lucas numbers are attached to Fibonacci numbers. Though the relationship between balancing and Lucas-balancing numbers is non-linear, like Fibonacci and Lucas numbers, they share the same linear recurrence $x_{n+1} = 6x_n - x_{n-1}$, while initial values of balancing numbers are $x_0 = 0,\ x_1 = 1$ and for Lucas-balancing numbers $x_0 = 1,\ x_1 = 3$. The objective of this talk is to present some Diophantine equations involving balancing and Lucas-balancing numbers and observe that the solutions of these equations are obtained in terms of balancing and Lucas-balancing numbers.

Keywords: Triangular numbers; Balancing numbers; Lucas-balancing numbers; Diophantine equations

References

[1] A. Behera and G.K. Panda, *On the square roots of triangular numbers*, The Fibonacci Quarterly, **37**(2) (1999), 98–105.

[2] B. Demirturk and R. Keskin, *Integer solutions of some Diophantine equations via Fibonacci and Lucas Numbers*, Journal of Integer Sequences, **12** (2009), Article 09.8.7.

[3] R. Keskin and O. Karaatl *Some New Properties of Balancing Numbers and Square Triangular Numbers*, Journal of Integer Sequences, **15** (2012), Article 12.1.4.

[4] K. Liptai, *Fibonacci balancing numbers*, The Fibonacci Quarterly, **42**(4) (2004), 330–340.

[5] G.K. Panda and P.K. Ray, *Cobalancing numbers and cobalancers*, International Journal of Mathematics and Mathematical Sciences, **2005(8)**, (2005), 1189–1200.

[6] G.K. Panda and P.K. Ray, *Some links of balancing and cobalancing numbers with Pell and associated Pell numbers*, Bulletin of the Institute of Mathematics, Academia Sinica (New Series), **6 (1)** (2011), 41–72.

[7] G.K. Panda, *Some fascinating properties of balancing numbers*, Proceeding Eleventh International Conference on Fibonacci Numbers and Their Applications, Cong. Numerantium, **194** (2009), 185–189.

[8] P.K. Ray, *Some congruences for balancing and Lucas-balancing numbers and their applications*, Integers **14** (2014), #A8.

Effective results for Diophantine equations over finitely generated domains

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Abstract

Let $A := \mathbb{Z}[z_1, \ldots, z_r] \supset \mathbb{Z}$ be a finitely generated integral domain over $\mathbb{Z}$ and denote by K the quotient field of A. Finiteness results for several kinds of Diophantine equations over A date back to the middle of the last century. S. Lang generalized several earlier results on Diophantine equations over the integers to results over A, including results concerning unit equations, Thue-equations and integral points on curves. However, all his results were ineffective.

The first effective results for Diophantine equations over finitely generated domains were published in the 1980's, when Győry developed his new effective specialization method. This enabled him to prove effective results over finitely generated domains of a special type.

In 2011 Evertse and Győry refined the method of Győry such that they were able to prove effective results for unit equations $ax + by = 1$ in $x, y \in A^*$ over arbitrary finitely generated domains A of characteristic 0. Later Bérczes, Evertse and Győry obtained effective results for Thue equations, hyper- and superelliptic equations and for the Schinzel-Tijdeman equation over arbitrary finitely generated domains.

In this talk I will present my effective results for equations $F(x, y) = 0$ in $x, y \in A^*$ for arbitrary finitely generated domains A, and for $F(x, y) = 0$ in $x, y \in \overline{\Gamma}$, where $F(X, Y)$ is a bivariate polynomial over A and $\overline{\Gamma}$ is the division group of a finitely generated subgroup Γ of K^*. These are the first effective versions of the famous corresponding ineffective results of Lang (1960) and Liardet (1974).

Keywords: diophantine equations; effective results

Diophantine equations concerning Appell sequences

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Abstract

The well-known examples of Appell sequences are the Bernoulli, Euler and Hermite polynomials. Diophantine equations involving these polynomial sequences have been considered by several authors.

In the talk we study the equation

$$P_n(x) = g(y) \tag{1}$$

in integers x, y where $P_n(x), g(x) \in \mathbb{Q}[x], \deg g(x) \geq 3$ and $\{P_n(x)\}_{n\geq 0}$ is an Appell sequence. In 2004, Rakaczki [4], and later independently Kulkarni and Sury [3] studied equation (1) with $P_n(x) = B_n(x)$ where $B_n(x)$ is the n-th Bernoulli polynomial. They proved a finiteness result on (1) which depended on the decomposability of $B_n(x)$. The results of Bilu, Brindza, Kirschenhofer, Pintér and Tichy [1] and of Kreso and Rakaczki [2] yield that the decomposition properties of the Bernoulli and Euler polynomials, respectively, are analogue. Motivated by this fact, we consider (1) for Appell sequences with the abovementioned decomposability properties. We present our ineffective finiteness result on (1) for this case.

Keywords: diophantine equations; Appell sequences; decomposition.

References

[1] Y. F. Bilu, B. Brindza, P. Kirschenhofer, Á. Pintér and R. F. Tichy, *Diophantine equations and Bernoulli polynomials (with an Appendix by A. Schinzel)*, Compositio Math. **131** (2002), 173–188.

[2] D. Kreso and Cs. Rakaczki, *Diophantine equations with Euler polynomials*, Acta Arith. **161** (2013), 267–281.

[3] M. Kulkarni and B. Sury, *On the Diophantine equation* $1 + x + \frac{x^2}{2!} + \ldots + \frac{x^n}{n!} = g(y)$., In: Diophantine equations, 121–134, *Tata Inst. Fund. Res. Stud. Math.*, Mumbai, 2008.

[4] Cs. Rakaczki, *On the Diophantine equation* $S_m(x) = g(y)$, Publ. Math. Debrecen, **65** (2004), 439–460.

Ke Zhao's method and its applications to Diophantine equations

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Abstract

Ke Zhao developed his method when he tried to solve the famous Catalan's Conjecture: The only solution of the diophantine equation

$$x^m - y^n = 1, x > 1, y > 1, m > 1, n > 1$$

is $(x, y, m, n) = (3, 2, 2, 3)$. The method is very powerful in solving the equations related to Lucas numbers. In this talk, I will show how this method works and will give its applications to the Diophantine equations of the form

$$Ax^4 - By^2 = c, \quad c \in \{\pm 1, \pm 2, \pm 4\}.$$

For example, we completely solved the diophantine equation $Ax^4 - By^2 = 2, 2 \not| AB$ in [2]. We also generalized this method in [1].

Keywords: Lucas numbers; quadratic residues; Diophantine equations.

References

[1] Pingzhi Yuan and Yongzhong Hu, *On the Diophantine equation $x^2 + D^m = p^n$*, J. Number Theory **111** (2005), no. 1, 144–153.

[2] Pingzhi Yuan and Yuan Li, *Squares in Lehmer sequences and the Diophantine equation $Ax^4 - By^2 = 2$*, Acta Arith. **139** (2009), no. 3, 275–302.

Perfect powers in a family of combinatorial polynomials

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Abstract

We investigate perfect powers in a family of polynomials defined by products of consecutive integers. We give general finiteness results, and also give all solutions when the number of terms in the sums considered does not exceed ten. To prove our theorems, we combine local arguments, results of Brindza on superelliptic equations, Baker's method, Runge's method, the theory of elliptic curves, and a method of Gebel-Pethő-Zimmer and Stroeker-Tzanakis to find all integral points on concrete elliptic equations.

The presented results are joint with S. Laishram and Sz. Tengely [1].

Keywords: combinatorial diophantine equations; blocks of consecutive integers; perfect powers

References

[1] L. Hajdu, S. Laishram and Sz. Tengely, *Power values of sums of products of consecutive integers*, Acta Arith. (accepted).

Orbits of rotations and beyond

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Abstract

The orbit of a rotation of the unit circle has one of two behaviours as a dynamical system, depending on whether the angle of rotation is a rational or an irrational multiple of 2π. In the rational case, any orbit is periodic; in the irrational case, any orbit is dense and in fact uniformly distributed. A quantitative form of the uniform distribution of the irrational orbits can be studied via the discrepancy of the sequence $\{n\alpha\}$ which in turn depends heavily on the continued fraction expansion of α.

The study of orbits of rotations leads naturally to the notion of twisted Diophantine approximation – a form of inhomogeneous Diophantine approximation, where one fixes the homogeneous parameter and studies the possible approximation properties of the inhomogeneous parameter as it varies. Many classical results in this field are to be found in the seminal paper of Kurzweil [3].

In [1], with Bugeaud, Harrap and Velani, we proved that the natural analogue of badly approximable elements exist in abundance. To be precise, we proved that for any $\alpha \in \mathbb{R}$, the set

$$\left\{x \in [0,1) : \|n\alpha - x\| \geq \frac{K(x)}{n} \text{ for some } K(x) > 0 \text{ for all } n \in \mathbb{N}\right\}$$

has maximal Hausdorff dimension and that this property is stable under intersection with sufficiently nice fractals. In recent work in progress with Tseng [2], this result is extended to the case when the rotation is replaced by an Interval Exchange Transformation (IET). The added difficulty of the IET setting comes from some highly geometric and dynamical obstacles to a sufficiently nice behaviour of the appropriate analogue of continued fractions.

Keywords: Irrational rotations; Twisted inhomogeneous Diophantine approximation; Interval Exchange Transformations

References

[1] Y. Bugeaud, S. Harrap, S. Kristensen and S. Velani, *On shrinking targets for $\mathbb{Z}^m$ actions on tori*, Mathematika **56(2)** (2010), 193–202.

[2] S. Kristensen and J. Tseng, *Badly approximable interval exchange transformations*, Work in progress.

[3] J. Kurzweil, *On the metric theory of inhomogeneous diophantine approximations*, Studia Math. **15** (1955), 84–112.

On the 2-part of the Birch and Swinnerton-Dyer conjecture for quadratic twists of elliptic curves

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Abstract

In my lecture, I shall discuss some joint work with J. Coates, Y. Li and Y. Tian, and some of my recent work on the 2-part of the Birch–Swinnerton-Dyer conjecture which applies rather classical results on modular symbols to the quadratic twists of a large family of elliptic curves over $\mathbb{Q}$. We prove that, for certain families of elliptic curves defined over $\mathbb{Q}$, there always exist a large class of explicit quadratic twists whose complex L-series does not vanish at $s = 1$. We also obtain a lower bound on 2-adic valuation of the algebraic part of the L-value at 1 in the family of quadratic twists of all optimal elliptic curves over $\mathbb{Q}$. Moreover, in some cases, we are able to prove the 2-part of exact Birch–Swinnerton-Dyer formula for some of these curves.

Keywords: BSD Conjecture; Quadratic Twists; Elliptic Curves.

References

[1] J. Coates, Y. Li, Y. Tian, S. Zhai, *Quadratic twists of elliptic curves*, Proc. Lond. Math. Soc. (3) **110** (2015), no. 2, 357–394.

[2] S. Zhai, *Non-vanishing theorems for quadratic twists of elliptic curves*, to appear in the Asian Journal of Mathematics.

[3] S. Zhai, *On the 2-part of the Birch and Swinnerton-Dyer conjecture for quadratic twists of elliptic curves*, to appear.

Diophantine m-tuples with values in linear recurrences

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Abstract

In this talk, we will survey existing results concerning Diophantine m-tuples with values in linear recurrences and present some new results. We shall start by mentioning a result obtained jointly with Fuchs and Szalay in 2008 which states that if $\{u_n\}_{n\geq 1}$ is a binary recurrent sequence satisfying certain conditions, then there are at most finitely many triples of positive integers $a < b < c$ such that $ab+1$, $ac+1$, $bc+1$ are all members of $\{u_n\}_{n\geq 1}$. The sequences of Fibonacci and Lucas numbers satisfy the conditions of the above theorem, and all Diophantine triples of positive integers with values in the Fibonacci sequence or the Lucas sequence were computed in joint work with Szalay in 2008 and 2009. Later Szalay and his co-authors extended the method to find all Diophantine triples with values in a certain parametric family of binary recurrent sequences which includes the Fibonacci sequence as a particular case. We shall also report on some new results concerning Diophantine triples and quadruples with values in the Tribonacci sequence, or the sequence of base b-repdigits, where $b \geq 2$ is any integer. Some of these results are joint with many colleagues such as Attila Bérczes (Hungary), Carlos Alexis Gómez Ruiz (Colombia), Clemens Fuchs (Austria), Bo He (China), Christoph Hutle (Austria), Nuretin Irmak (Turkey), Istvan Pink (Hungary), Laszlo Szalay (Hungary) Alain Togbé (USA), and Volker Ziegler (Austria).

Keywords: Diophantine m-tuples; Linearly recurrent sequence

References

[1] Attila Bérczes, Florian Luca, István Pink and Volker Ziegler, *Finiteness results for Diophantine triples with repdigit values*, arXiv:1502.05550.

[2] Clemens Fuchs, Christoph Hutle, Nurettin Irmak, Florian Luca and Laszlo Szalay, *Only finitely many Tribonacci Diophantine triples exist*, arXiv:1508.07760.

[3] Carlos Alexis Gómez Ruiz and Florian Luca, *Tribonacci Diophantine Quadruples*, *Glasnik Math.* **50(70)** (2015), 17–24.

[4] Bo He, Florian Luca and Alain Togbé, *Diophantine triples of Fibonacci numbers*, Preprint, 2015.

On recurrence-based numerations

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Abstract

If $A = (a_n)_{n\geq 0}$, $(a_0 = 1)$, is an increasing integral linear recurrence with characteristic polynomial

$$f(x) = x^m - P_1 x^{m-1} - P_2 x^{m-2} - \cdots - P_m,$$

where the P_i's are nonnegative integers, then every integer can be uniquely represented, using the greedy algorithm, as a sum $\sum_{i=0}^{k} d_i a_i$. If the coefficients satisfy $P_1 \geq P_2 \geq \cdots \geq P_m \geq 1$, then the cumulative sum-of-digit function $S_A(n) := \sum_{k<n} s_A(k)$, where $s_A(n) = \sum_i d_i$ is the digit sum of the integer $n = \sum_{i=0}^{k} d_i a_i$, is well-known to satisfy

$$S_A(n) = c_A n \log n + O(n). \qquad (2)$$

While much work has been carried out [6, 3, 2, 5, 4] on the remainder term $O(n)$, which for some recurrences such as the Fibonacci, or the tribonacci numbers, takes the form $nG(\log n/\log\alpha) + O(\log n)$ with G of period one, continuous and nowhere differentiable and α is the dominant zero of f, we have recently been interested in establishing (2) for a wider class of recurrences and trying to find out the least constant c_A within our class for all f of a given degree [1].

This study singled out a family of recurrences, one per characteristic polynomial $x^q - x^{q-1} - 1$ for each $q \geq 1$. We will also provide some relationships this family of recurrences bears with compositions, binomial coefficients and Nim games.

Our work is elementary. Other particular recurrence-based numerations will be presented.

Keywords: numeration; recurrence; average digit sum

References

[1] C. Ballot, *On the constant in the average digit sum for a recurrence-based numeration*, to appear in Unif. Distrib. Theory.

[2] J. Coquet and P. van den Bosch, *A summation formula involving Fibonacci digits*, J. Number Theory **22** (1986), 139–146.

[3] H. Delange, *Sur la fonction sommatoire de la fonction 'Somme des Chiffres'*, Enseignement Math. **21** (1975), 31–47.

[4] P. J. Grabner and R. F. Tichy, *Contributions to digit expansions with respect to linear recurrences*, J. Number Theory **36** (1990), 160–169.

[5] A. Pethö and R. F. Tichy, *On digit expansions with respect to linear recurrences*, J. Number Theory **33** (1989), 243–256.

[6] J. R. Trollope, *An explicit expression for binary digital sums*, Math. Mag. **41** (1968), 21–25.

Commom difference sets, higher order recurrence sets and related problems

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Abstract

Commom difference sets, higher order recurrence sets and related problems

Abstract: It is believed that a ˆ big ¨ subset of natural numbers should contain ˆ good ¨ linear structures, for example arbitrarily long arithmetic progressions (AP). Szemerdi's Theorem asserts that every positive upper Banach density subset of natural numbers has this property. In this talk, we will consider the structure of all common differences of AP with length k+1 appeared in a ˆ big ¨ subset of natural numbers. By the Furstenberg correspondence principle, the common difference set of AP of a syndetic (or positive upper Banach density) subset are related to the higher order recurrence sets in dynamical systems. It is shown that a higher order recurrence sets is an almost Nil Bohr-sets, and Nil Bohr-sets could be characterized via generalized polynomials. Hence the common difference set of AP of a syndetic (or positive upper Banach density) subset is an almost Nil Bohr-sets, which could be characterized via generalized polynomials. Some related problems in dynamical system (for example multiple ergodic average problem) will be also discussed. Finally we will review some progress on the difference set of primes.

Keywords: Common differenc sets; Nil Bohr set; generalized polynomial; multiple ergodic average problem.

Extremal problems for distances and sum-products

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Abstract

A classical problem in the field of discrete geometry is the Erdős distinct distance problem, which asks what the minimum possible number of distinct distances determined by a planar point set is. The problem has now been essentially resolved by Guth and Katz, but the problem of determining which sets determine the fewest distinct distances remains wide open

This talk aims to introduce this problem and state some non trivial results which utilise number theoretic tools. The connection with some similar extremal sum-product type problems will also be discussed.

Keywords: Erdős distance problem, sum-product estimates, extremal questions

Minor arcs estimate in the application of circle method

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Abstract

The Hardy-Littlewood circle method plays an important role in the study of Diophantine equations. The minor arcs type estimate is a crucial step in the application of circle method. In this talk, we will introduce some new development on the minor arcs type estimate.

Keywords: Diophantine equations; circle method; minor arcs.

On the saturation number for cubic surfaces

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Abstract

We generalize the definition of the saturation number introduced by Bourgain, Gamburd and Sarnak to investigate the distribution of rational points on smooth cubic surfaces whose coordinates have few prime factors. Let S be a smooth cubic surface defined over $\mathbb{Q}$, embedded in projective space $\mathbb{P}^3$. Define the saturation number $r(S)$ to be the least number r such that the set of $\mathbf{x} = (x_0, x_1, x_2, x_3) \in \mathbb{Z}^4_{\text{prim}}$ for which $[\mathbf{x}] \in S$ and the product $x_0x_1x_2x_3$ has at most r prime factors, is Zariski dense in S. In this talk, we establish the finiteness of the saturation number for all smooth cubic surfaces containing a rational point. Our approach is based on unirationality arguments in combination with the weighted sieve. This is joint work with E. Sofos (Leiden University).

Keywords: cubic surfaces; weighted sieve; unirational.

Number of solutions for sextic simple Thue equations

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Abstract

For $i = 3, 4, 6$ there is the so-called simple family $F_{a,b}^{(i)}(X,Y)$ of degree i. We consider the number of solutions for the simple Thue equation $F_{a,b}^{(i)}(x,y) = 1$ (or ± 1). We showed in [1] that for cubic case this number is 0 or 3 except for a known few cases, and in [2] that for quartic case this number is 0 or 4 except for a known few cases. As a continuation of the research, we consider the sextic case. Let $F(X,Y) = F_{a,b}^{(6)}(X,Y) = bX^6 - 2aX^5Y - (5a+15b)X^4Y^2 - 20bX^3Y^3 + 5aX^2Y^4 + (2a+6b)XY^5 + bY^6 \in \boldsymbol{Z}[X,Y]$. We show that the number of solutions for the sextic Thue equation $F(x,y) = \pm 1$ is 0 or 6. For the proof, we obtain an upper bound for the size of solutions by Padé approximation method. Then, by comparing the upper bound with a lower bound for the size of solutions which can be derived elementarily, we obtain the result except for a finite number of pairs (a,b). For the remaining pairs (a,b), we solve the equation by computer, and obtain the result.

Keywords: simple family; Thue equation; number of solutions

References

[1] I. Wakabayashi, *Number of solutions for cubic Thue equations with automorphisms*, Ramanujan J. **14** (2007), 131–154.

[2] I. Wakabayashi, *Number of solutions for quartic simple Thue equations*, International J. Number Theory **8** (2012), 1367–1386.

Sums and reciprocal sums involving balancing numbers

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Abstract

A balancing number is a natural number n that satisfy the Diophantine equation $1+2+\cdots+(n-1)=(n+1)+\cdots+(n+r)$ for some natural number r, called the balancer corresponding to n [1]. The balancing numbers satisfy the binary recurrence $B_{n+1}=6B_n-B_{n-1}$ with initial values $B_0=0$ and $B_1=1$. They constitute a divisibility sequence and enjoy many fabulous properties. Similar class of sequences defined recursively by $x_{n+1}=Nx_n-x_{n-1}$ with initial values $x_0=0$ and $x_1=1$ ($N>2$ is any natural number) are also divisibility sequences and enjoy properties similar to balancing numbers and hence are known as balancing-like sequences. Panda in [5] established the sum formulas $B_1+B_3+\cdots+B_{2n-1}=B_n^2$ and $B_2+B_4+\cdots+B_{2n}=B_nB_{n+1}$. In a recent paper [2], several sum formulas involving balancing numbers were developed by Davala and Panda. Holliday and Komatsu [3] studied reciprocal sums for generalized Fibonacci numbers. Melham contributed a lot to the study on reciprocal sums involving Fibonacci numbers and in his recent contribution [4], he studied these sums involving generalized Fibonacci numbers. The objective of this talk is to discuss certain sum formulas and bounds for sums involving reciprocals of balancing and balancing-like numbers.

Keywords: Balancing sequence; balancing-like sequences; reciprocal sum

References

[1] A. Behera and G. K. Panda, *On the square roots of triangular numbers,* Fib. Quart., **37(2)** (1999), 98-105.

[2] R.K. Davala, G.K. Panda *On sum and ratio formulas for balancing numbers*, J. Indian Math. Soc. **82** (2015), 23-32.

[3] S.H. Holliday, T. Komatsu, *On the sum of reciprocal generalized Fibonacci numbers*, Proceedings of the Integers Conference 2009, 11A.

[4] R.S. Melham, *On Certain Families of Finite Reciprocal Sums that Involve Generalized Fibonacci Numbers*, Fibonacci Quart., **53** (2015), 323–334.

[5] G.K. Panda, *Some fascinating properties of balancing numbers,* In Proc. of Eleventh Internat. Conference on Fibonacci Numbers and Their Applications, Cong. Numerantium, **194** (2009), 185–189.

Pascal-like arrays and triangles

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Abstract

First we give an overview on the generalization possibilities and the extensions of Pascal's original arithmetical triangle. Among them we pay more attention to the Fibonacci, and Lucas Pascal triangles, further hyperfibonacci numbers. In the last case, we investigate the common values among them. Then we study new objects, the so-called hyperbolic Pascal triangles.

Keywords: Pascal triangles; Arithmetical triangles

References

[1] H. Belbachir, T. Komatsu, L. Szalay, *Linear recurrences associated to rays in Pascal's triangle and combinatorial identities*, Math. Slovaca, **64** (2) (2014), 287-300.

[2] H. Belbachir, L. Németh, L. Szalay, *Hyperbolic Pascal triangles*, accepted in Appl. Math. Comp.

[3] H. Belbachir, L.Szalay, *On the arithmetic triangles*, Šiauliai Math. Sem., **9** (17) (2014), 15–26.

[4] A. Dil, I. Mező, *A symmetric algorithm for hyperharmonic and Fibonacci numbers*, Appl. Math. Comp., **206** (2008), 942-951.

[5] D. Ensley, *Fibonacci's triangle and other abominations*, in: The Edge of the Universe: Celebrating Ten Years of Maths Horizons, MAA, 287-307, 2006.

[6] B. Pascal, *Traité du Triangle Arithmétique, avec quelques autres petitez sur la mesme matiere*, Paris, 1665.

Partitions without small blocks

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Abstract

We study the log-concavity property of the number of partitions of an n element set into a given number of non-singleton blocks through the real zero property of the generating polynomials. We locate the asymptotic behavior of the peak.

In the second part of the paper we discuss the asymptotic behavior of the number of partitions of an n-set into a given number of "not too small" blocks.

Keywords: partitions; log-concavity; asymptotics

References

[1] M. Bóna and I. Mező, *Real zeros and partitions without singleton blocks*, Eur. J. Combin., **51** (2016), 500–510.

[2] C.-Y. Wang and I. Mező, *Some limit theorems with respect to constrained permutations and partitions*, submitted.

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